Chord length sampling correction analysis for dispersion

fuel in Monte Carlo simulation*

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Abstract

Due to their excellent safety performance, dispersion fuels are widely used in advanced reactors, such as high-temperature gascooled reactors (HTGRs). Compared with deterministic methods, the Monte Carlo method has more advantages in the geometric
modeling of stochastic media. The explicit modeling method has high computational accuracy and high computational cost. The
chord length sampling (CLS) method can improve computational efficiency by sampling the chord length during neutron
transport using the matrix chord length's probability density function (PDF). This study shows that the excluded-volume effect in
realistic stochastic media can introduce significant deviations into the CLS. A chord length correction approach is proposed to
obtain the chord length correction factor by developing the Particle code based on equivalent transmission probability. Through
numerical analysis against reference solutions from explicit modeling in the RMC code, it was demonstrated that CLS with the
proposed correction method provides good accuracy for addressing the excluded-volume effect in realistic infinite stochastic
media.

Keywords: Stochastic media; Monte Carlo; Chord Length Sampling; Excluded-volume effect; Chord Length Correction

1. Introduction

Dispersion fuel can effectively prevent nuclear leakage, has high safety performance, and is widely used in advanced reactors, particularly high-temperature gas-cooled reactors (HTGRs). In traditional deterministic methods, the presence of stochastic media causes the dispersed fuel to exhibit double heterogeneity, which poses a challenge to reactor physics calculations. The Monte Carlo method has significant advantages in the simulation of stochastic media due to its flexible geometric modeling and continuous energy nuclear cross-sections. Several approaches have been developed to simulate stochastic media by considering stochastic effects, such as random explicit modeling and the chord length sampling (CLS) method.

^{*} This work is supported by the CNNC Youth Research Project.

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The explicit modeling method can accurately describe the geometry of each grain, depending on the method of generating stochastic grains. Early studies [1-3] placed grains in regular lattices to describe stochastic media without considering the randomness of the grain distribution. Brown et al. [4] introduced random translations into the positions of grains within repeated regular lattices and implemented them in MCNP. This random lattice method considers some randomness; however, the grains remain relatively regular at the macro level. The explicit random modeling method can accurately describe each stochastic grain with the least approximation based on random packing methods such as Random Sequential Addition (RSA) [5], Close Random Packing (CRP) [6] and Discrete Element Method (DEM) [7]. The generation and storage of grains and the ray tracing process incur significant computational costs. Therefore, the random explicit modeling method with the highest accuracy is often used to provide reference solutions and has been used in Monte Carlo codes such as OpenMC [8], SERPENT [9] and RMC [10, 11].

Unlike explicit methods, an implicit method called CLS samples the matrix chord-length and entry angle to reach the next grain during neutron transport without explicitly describing the stochastic media. Zimmerman and Adams [12] proposed the CLS method based on 1D geometry. Murata et al. [13, 14] developed an MCNP-CFP code to simulate HTGR based on the nearest neighbor distribution (NDD) method. CLS has the advantage of simplifying geometric modeling and improving computational efficiency.

The CLS method uses the matrix chord length's probability density function (PDF) to sample the distance to the next grain, indirectly determining the grains' packing fraction. The PDF of matrix chord length is inherently assumed to satisfy a Poisson distribution [15-18], because the transport process of CLS is a Markovian process in which previous grains do not affect future grains (i.e., overlapping between grains is not considered). In realistic stochastic media, the chord length distribution deviates from the Poisson distribution, influenced by the boundary effect of the external boundary and excluded-volume effect of the micro grains.

A major problem in finite stochastic media is the boundary effect, which reduces the local packing fraction near an external boundary to avoid overlap between grains and external boundary. Thus, the total packing fraction is lower than expected. To address the boundary effect, many studies have focused on correction methods for the packing fraction influenced by finite boundaries, such as those by Murata [13], Ji [15], Griesheimer [19, 20] and Liang [21]. These studies were based on the assumption that the PDF follows an exponential distribution, and focused on the methods for correcting the packing fraction to balance the boundary effect.

The excluded-volume effect is an existing but unresolved problem in realistic stochastic media. To avoid overlapping between grains, the former grains have an excluded-volume effect on the latter grains during packing, resulting in a significant decrease in the proportion of matrix chord length near the grain size. The excluded-volume effect can cause the matrix chord length to deviate from the exponential distribution. However, this effect has been neglected in previous studies because the grain size is usually sufficiently small to ignore the interactions between grains, especially at low packing fractions. The excluded-volume effect conflicts with the Poisson distribution assumption of the CLS, making it difficult to correct.

In this study, we analyzed the mechanism of the excluded-volume effect on the matrix chord length PDF. Subsequently, a Particle code was developed to generate stochastic grains rapidly and perform chord length sampling analysis. Furthermore, a chord-length correction method was proposed based on the matrix transmission probability equivalence to obtain the matrix chord length correction factor. The CLS method with correction was implemented in the RMC code [10, 11, 22]. Cases with infinite stochastic media were used to verify the accuracy of the modified CLS method while avoiding the influence of boundary effects.

The remainder of this paper is organized as follows: Section 2 introduces the basic principles and excluded-volume effect issues of the CLS method, and a correction approach for the chord-length is proposed based on the Particle code. The verification of the CLS with chord length correction is presented in Section 3. Finally, the conclusions are presented in Section 4.

2. Methodology

2.1 Chord Length Sampling

There may be some differences in the implementation of CLS for different Monte Carlo codes. The CLS process in the RMC code [23] is shown in Fig. 1. After entering the stochastic media region, the neutrons pass through the matrix and particles until they experience their first collision. A characteristic of CLS is using a PDF to describe the matrix chord-length, which is the length of the matrix path traveled by neutrons leaving the current grain to reach the next grain. The type and position of the next grain were determined by sampling.

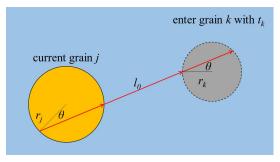


Fig. 1. Schematic diagram of CLS

Firstly, the matrix chord-length l_{θ} between the grains is sampled based on the PDF in Eq. .

$$p(l_0) = \frac{1}{l_0} e^{-\frac{l_0}{l_0}} \tag{1}$$

Here, the PDF is assumed exponential, and the matrix mean chord-length $\overline{l_0}$ is defined by Eq. .

$$\frac{1}{l_0} = \frac{4}{3} \frac{1 - \sum_{j} f(j)}{\sum_{j} \frac{f(j)}{r_j}}$$
(2)

This matrix mean chord length is different from the traditional CLS because it considers the case of a stochastic media containing multiple grains, such as fuel and poison grains, where f(j) represents the packing fraction of grain j with radius r_j .

Secondly, the type of the next grain entering is sampled, entering grain j with probability t_i using Eq. .

$$t_{j} = \frac{f(j)}{r_{i}} / \sum_{m} \frac{f(m)}{r_{m}}$$
(3)

Thirdly, based on the assumption that neutrons enter grains isotropically, the angle θ in which the neutron enters the next sphere is sampled, and then the spherical center of the next grain is determined. The angle θ can be sampled by its cosine value $\mu = cos(\theta)$ from Eq. .

$$\mu = \sqrt{\xi} \tag{4}$$

Where, ξ is a random number between (0, 1).

2.2 Excluded-volume effect

The excluded-volume effect is a problem in realistic stochastic media. Because of the hard-sphere assumption, grains cannot overlap with other grains. The excluded-volume effect impacts both the explicit modeling and CLS methods.

In the explicit modeling method, owing to the hard-sphere assumption, the newly sampled grain is rejected and resampled when overlapping with any existing grain, resulting in a matrix chord-length that is not equal to 0. When the matrix chord-length is extremely small and close to zero, the number of small chord lengths decreases significantly, causing the matrix chord length distribution to deviate from the exponential distribution, as shown in Fig. 2.

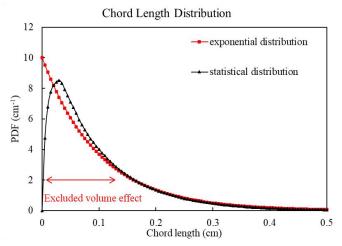


Fig. 2. Schematic diagram of matrix chord length distribution in realistic stochastic media influenced by excluded-volume effect. In the CLS method, the physical process is assumed to be a Markov process, in which the newly sampled grains do not need to consider the previous grains, and the PDF follows an exponential distribution. This physical process conflicts with the correction method that considers the overlap between grains. Therefore, correcting the PDF for the excluded-volume effect is difficult.

We noticed that a correction treatment is used in some codes to determine whether the grains overlap, considering that the next grain may overlap with the current grain when the chord length is small, as shown in

Fig. 3. When an overlap occurs between grains, the current sampling result is rejected, and the chord-length and angle are resampled.

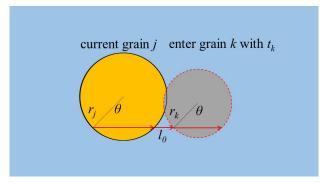


Fig. 3. Next grain overlapping with the current grain

This correction treatment is unnecessary, although it appears more consistent with realistic stochastic media. The physical process of CLS is a Markov process in which neutrons leave the current grain and enter the matrix without considering the history of the previous grains. In other words, there is no need to consider the overlapping problem of previous grains in the CLS. Suppose it is necessary to consider the overlap between grains; not only the previous one but also all grains in history should be considered, which violates the principle of the CLS. This correction causes many smaller chord lengths to be rejected, whereas larger ones are retained, increasing the mean chord-length, which is equivalent to a reduction in the packing fraction of the grains.

2.3 Particle code

The Particle code was developed to quickly model realistic stochastic grains and sample matrix chord lengths to address the excluded-volume effect. As shown in Fig. 4, the implementation of the Particle code includes the following processes.

- (1) The geometry of a finite region was defined, and a virtual acceleration mesh was established around the entire geometry.
- (2) With the virtual grid acceleration, RSA and CRP are combined to quickly generate the coordinates of stochastic grains while establishing an index between grains and the virtual grid.
- (3) Randomly sample the matrix chord length to the entry grain and angle based on ray tracing and white boundary condition, search for the nearest grain on the path, and record the chord-length information. This chord length sampling process was repeated N times to obtain the distribution of the matrix chord lengths.
- (4) To reduce the random error, we replace the random seed and repeat (1) (3) to obtain a stable statistical result.

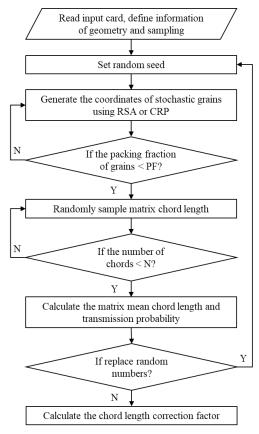


Fig. 4. The flowchart of the Particle code

The Particle code is based on geometric chord length sampling of realistic stochastic media without considering real nuclear physics reactions. The calculation time of the Particle code is much shorter than that of the Monte Carlo neutron transport simulation based on explicit modeling. Therefore, its simulation efficiency is very high, usually taking a few seconds under one core at a low packing fraction, which can be almost ignored.

2.4 Chord length correction

The Particle code is based on geometric chord length sampling of realistic stochastic media, and it is not appropriate to use the statistical matrix mean chord-length in the exponential distribution directly after obtaining the matrix chord length distribution because it no longer satisfies the exponential distribution. The matrix chord length corresponds to the matrix transmission probability for a physical process in the stochastic media. Therefore, a chord length correction method based on the equivalent transmission probability is proposed, which ensures the conservation of the matrix transmission probability by correcting the matrix chord length. This correction method has been implemented in the chord-length-based double heterogeneity model in the XPZ code [24] and has shown high accuracy and application prospects.

After the chord length sampling procedure in the Particle code is completed, the mean transmission probability of the matrix is calculated using Eq..

$$T_0^{MC} = \frac{1}{N} \sum_{i}^{N} e^{-\Sigma_0 t_0^{(i)}}$$
 (5)

And, the matrix mean chord-length $\overline{l_{\scriptscriptstyle 0}}^{\scriptscriptstyle -MC}$ is calculated by Eq..

$$\overline{l_0}^{MC} = \frac{1}{N} \sum_{i}^{N} l_0^{(i)} \tag{6}$$

Where $I_0^{(i)}$ is the sampling chord length with index i among N sampling results.

Graphite or SiC is commonly chosen as the matrix material for dispersion fuels in HTGRs, compact gascooled reactors, or FCM ^[25] fuel in PWRs, and the cross-section is usually insensitive to energy. The cross-section can be discretized into a multigroup problem based on energy if it is sensitive to energy. It is assumed that the macroscopic total cross-section of the matrix material is a constant value within an energy range. Assuming an exponential distribution, the theoretical matrix transmission probability can be expressed using Eq. .

$$T_{0} = \int_{0}^{\infty} e^{-\Sigma_{0} l_{0}} \frac{1}{e^{-\frac{l_{0}}{l_{0}}}} dl_{0} = \frac{1}{1 + \Sigma_{0} l_{0}}$$
 (7)

By equating T_0 with T_0^{MC} , the corrected chord-length \overline{l}_0^{-Eq} can be derived using Eq. .

$$\overline{l_0}^{Eq} = \frac{1 - T_0^{MC}}{T_0^{MC} \Sigma_0}$$
 (8)

The boundary effect is unavoidable when the particle code generates stochastic grains. Therefore, stochastic grains should be generated in the largest possible region until the statistical matrix mean chord length $\overline{l_0}^{\text{\tiny MC}}$ approaches the theoretical value obtained using Eq. . Finally, a dimensionless matrix chord length correction factor is defined by Eq. , which is used to correct the matrix mean chord-length under the assumption of an exponential distribution.

$$C = \frac{\overline{l_0}^{Eq}}{\overline{l_0}} \tag{9}$$

This factor is multiplied by the sampled chord length to obtain the corrected chord length in the CLS. Because the excluded-volume effect is considered in the Particle code's modeling procedure, the corrected CLS is more consistent with the explicit modeling method in infinite stochastic media.

Additionally, to improve computational efficiency, an interpolation table can be established in advance to avoid calling the external code. We found that the chord length correction factor was correlated with the grain radius r and packing fraction f; thus, it can be established as shown in Table 1. The macroscopic total cross-section Σ_0 was set to 0.412/cm in the full energy range when the matrix material was graphite with a 1.73g/cm^3 density.

Table 1. The interpolation table of chord length correction factor										
Packing	1%	5%	10%	20%	30%					
fraction										
Radius (µm)										
250	1.0051	1.0046	1.0044	1.0038	1.0029					
500	1.0098	1.0093	1.0089	1.0077	1.0059					
750	1.0154	1.0137	1.0132	1.0112	1.0087					

Where r is the radius of the grain (cm), and f is the packing fraction. This formula is suitable for use in specific problems, such as the fuel element of the HTGR, and requires reprocessing for new problems.

3. Numerical results and analysis

3.1 Test Case

The CLS and its correction methods mentioned in this paper were implemented in the RMC code using efficient computational convergence methods ^[26]. A set of infinite stochastic media cases was designed to validate the effectiveness of the proposed method in addressing the excluded-volume effect. The numerical results from explicit modeling in the RMC were used as reference solutions and compared with the CLS method. The same cross-section from the ENDF/B-VIII.0 nuclear data library was adopted for the calculation in the RMC to ensure a direct comparison of the transport methods. The RMC uses settings of 40,000 particles, 2,000 generations, and 50 inactive generations to reduce the standard deviation of the infinite multiplication factor (k_{inf}) to within 10 pcm.

The infinite stochastic media shown in Fig. 5 is simplified from the fuel region of the standard fuel pebble in the HTR-10 and HTR-PM reactors $^{[27,28]}$. The matrix of an infinite stochastic media is graphite and fuel grains are randomly dispersed within it, with a UO₂ kernel outer radius of 250 μ m and a graphite coating layer outer radius of 455 μ m. The applicability of different neutron energy spectrum was verified by changing the grain packing fraction from 1% to 30%.

The chord length correction coefficient C was calculated using the Particle code where 1 000 000 chords were sampled, the matrix macroscopic total cross-section was set to 0.412/cm, and the boundary radius was greater than 7.5 cm until the results converged.

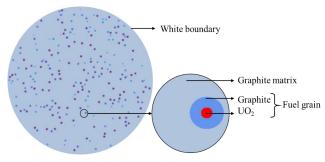


Fig. 5. Simplified infinite stochastic media by explicit modeling

3.2 Sensitivity analysis

Because the CLS method does not require explicit modeling of grains, an external boundary with an infinite radius was set to calculate k_{inf} . However, the explicit modeling of infinite stochastic media is unrealistic.

Therefore, a finite region of stochastic media with white boundary condition was used to simulate an infinite stochastic media. To minimize the boundary effect, the sensitivity of k_{inf} was studied by increasing the outer radius of the finite region. The radius of the stochastic media was selected as 0.5, 1.5, 2.5, 5.0, 7.5, and 10.0 cm, and the result for 10.0 cm was taken as the reference.

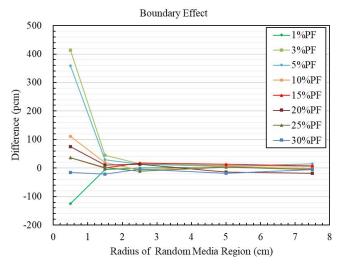


Fig. 6. Sensitivity Curve of Boundary Effect

The difference curve of k_{inf} with the radius for the various packing fractions (PFs) is shown in Fig. 6. It can be observed that there is a significant difference between the reference solution and the calculated result, with a strong boundary effect at a boundary radius of 0.5 cm. Furthermore, when the explicitly modeled region was small, the proportion of the region near the boundary with a lower packing fraction increased. In contrast, the other region far from the boundary had a larger packing fraction. This leads to a more uneven spatial distribution of the packing fraction and an enhanced boundary effect, causing the calculated result to deviate from the infinite reference solution. As the radius of the external boundary increased, the boundary effect weakened, whereas the deviation of k_{inf} decreased. After the radius exceeds 2.5 cm, the deviation can be controlled within \pm 20 pcm, indicating that the external boundary has almost no effect on the stochastic media. Stochastic media with radius of 10 cm can be used as reference solution for infinite stochastic media.

3.3 Numerical results

Table 2 compares the numerical results of the explicit modeling and CLS methods. First, the numerical results of the original CLS method are not significantly different from those of the explicit modeling, with a maximum of -170 pcm, which is acceptable in engineering but slightly larger for Monte Carlo simulations. In more complex finite stochastic media, the deviation is amplified when the excluded-volume effect is not considered.

The matrix chord length correction method for the excluded-volume effect is listed in Table 2. The chord length correction factor, C, was obtained using Eq. . It can be observed that the CLS method with chord length correction is more consistent with explicit modeling than the original CLS method. Across all calculated packing fractions, the deviations are within ± 50 pcm, indicating that the chord length correction method

proposed in this paper can effectively reduce the influence of the excluded-volume effect. The micro heterogeneity of the local stochastic grains is the main reason for the minor differences in k_{inf} .

Packing fraction	Evaliait modeling	CLS					
	Explicit modeling	Original		Equivalent correction			
	k_{inf}	k_{inf}	$\Delta k_{inf}(pcm)$	k_{inf}	$\Delta k_{inf}(\text{pcm})$	C	
0.01	1.74386	1.74412	26	1.74352	-35	1.00892	
0.03	1.68906	1.68813	-92	1.68942	37	1.00867	
0.05	1.58367	1.58230	-137	1.58416	49	1.00843	
0.10	1.39134	1.38964	-170	1.39180	46	1.00782	
0.15	1.27505	1.27342	-163	1.27518	13	1.00720	
0.20	1.20278	1.20161	-117	1.20258	-20	1.00659	
0.25	1.15726	1.15611	-114	1.15689	-37	1.00598	
0.30	1.12860	1.12764	-96	1.12818	-42	1.00537	

It should be noted that the accuracy of this correction method depends on the chord length sampling statistical method and the matrix's cross-section. If the scale of chord length sampling is increased or the cross-section is discretized according to energy to calculate the chord length correction factor. In that case the results will be more consistent with explicit modeling.

4. Conclusions

CLS is an efficient implicit method for treating dispersion fuels with stochastic media in Monte Carlo simulations. A basic assumption of CLS is the Markov process without considering the excluded-volume effect between grains. This study highlighted the excluded-volume effect of realistic stochastic media and discussed its impact on the matrix chord length distribution. A chord length correction method was proposed based on the equivalent transmission probability to eliminate this effect. Moreover, the Particle code with stochastic grain modeling and chord length sampling capabilities was developed to obtain the chord length correction factor. The corrected CLS method proposed in this study can be easily implemented using existing CLS-based Monte Carlo codes. Finally, with respect to the excluded-volume effect, cases of infinite stochastic media were established to verify the correction method. Numerical comparative analysis with explicit modeling has proven that the chord-length correction method proposed in this study can overcome the defects of neglecting the excluded-volume effect between grains in the original CLS method and has higher accuracy. In future work, we will focus on studying a correction method that considers both the boundary effect and the excluded-volume effect in realistic dispersion fuels with finite stochastic media based on previous works on boundary effect.

ACKNOWLEDGEMENTS

This work is supported by the National S&T Major Project (grant No. ZX06901/ZX06902) of China, and the CNNC Youth Research Project.

Author contributions:

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Zhaoyu Liang, Ding She, Yutong Wen and Lei Shi. The first draft of the manuscript was written by Zhaoyu Liang and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

References

- R. Plukiene, D. Ridikas, Modelling of HTRs with Monte Carlo: from a homogeneous to an exact heterogeneous core with microparticles. Ann. Nucl. Energy 30, 1573-1585 (2003). doi:10.1016/S0306-4549(03)00101-4
- 2. F.C. Difflippo, Monte Carlo Calculations of Pebble Bed Benchmark Configurations of the PROTEUS Facility. Nucl. Sci. Eng. **143**, 240-253 (2003). doi:10.13182/NSE02-34
- 3. W. Ji, J.L. Conlin, W.R. Martin, J.C. Lee, Reactor physics analysis of the VHTGR core. Transactions of the American Nuclear Society. **91**, 556-558 (2004).
- 4. F. Brown, W. Martin, Stochastic geometry capability in MCNP5 for the analysis of particle fuel. Ann. Nucl. Energy **31**, 2039-2047 (2004). doi:10.1016/j.anucene.2004.08.006
- 5. B. Widom, Random Sequential Addition of Hard Spheres to a Volume. J. Chem. Phys. 44, 3888-3894 (1966). doi:10.1063/1.1726548
- 6. W.S. Jodrey, E.M. Tory, Computer simulation of close random packing of equal spheres. Phys. Rev. A **32**, 2347-2351 (1985). doi:10.1103/PhysRevA.32.2347
- 7. A.M. Ougouag, J.J. Cogliati, J.L. Kloosterman. Methods for modeling the packing of fuel elements in pebble bed reactors. Paper presented at Mathematics and Computation, Supercomputing, Reactor Physics and Nuclear and Biological Applications, American Nuclear Society, Avignon, France, 12-15 Sept 2005.
- 8. P.K. Romano, N.E. Horelik, B.R. Herman et al., OpenMC: A state-of-the-art Monte Carlo code for research and development. Ann. Nucl. Energy **82**, 90-97 (2015). doi:10.1016/j.anucene.2014.07.048
- 9. J. Leppänen. Serpent A Continuous-Energy Monte Carlo Reactor Physics Burnup Calculation Code, User's Manual. (2015).
- 10. S.C. Liu, D. She, J.G. Liang et al., Development of random geometry capability in RMC code for stochastic media analysis. Ann. Nucl. Energy **85**, 903-908 (2015). doi:10.1016/j.anucene.2015.07.008
- 11. S.C. Liu, Z.G. Li, K. Wang et al., Random geometry capability in RMC code for explicit analysis of polytype particle/pebble and applications to HTR-10 benchmark. Ann. Nucl. Energy **111**, 41-49 (2018). doi:10.1016/j.anucene.2017.08.063
- 12. G.B. Zimmerman, M.L. Adams, Algorithms for Monte Carlo particle transport in binary statistical mixtures. Transactions of the American Nuclear Society **63**, 287-288 (1991).

- 13. I. Murata, T. Mori, M. Nakagawa, Continuous energy Monte Carlo calculations of randomly distributed spherical fuels in high-temperature gas-cooled reactors based on a statistical geometry model. Nucl. Sci. Eng. **123**, 96-109 (1996). doi:10.13182/NSE96-A24215
- I. Murata, A. Takahashi, T. Mori et al., New Sampling Method in Continuous Energy Monte Carlo Calculation for Pebble Bed Reactors. J. Nucl. Sci. Technol. 34, 734-744 (1997). doi:10.1080/18811248.1997.9733737
- 15. W. Ji, W.R. Martin. Application of chord length sampling to VHTR unit cell analysis, Paper presented at International Conference on the Physics of Reactors (PHYSOR 2008), American Nuclear Society, Interlaken, Switzerland 14-19 Sept 2008.
- 16. S. Torquato, B. Lu, Chord-length distribution function for two-phase random media. Phys Rev E **47**, 2950-2953 (1993). doi:10.1103/physreve.47.2950
- T.J. Donovan, Y. Danon, Application of Monte Carlo chord-length sampling algorithms to transport through a two-dimensional binary stochastic mixture. Nucl. Sci. Eng. 143, 226-239 (2003). doi:10.13182/NSE03-A2332
- 18. B.L. Lu, S. Torquato, Lineal-path function for random heterogeneous materials. Phys. Rev. A **45**, 922-929 (1992). doi:10.1103/PhysRevA.45.922
- 19. D.P. Griesheimer, D.L. Millman, C.R. Willis, Analysis of distances between inclusions in finite binary stochastic materials. J. Quant. Spectrosc. Radiat. Transf. **112**, 577-598 (2011). doi:10.1016/j.jqsrt.2010.06.013
- D.P. Griesheimer, D.L. Millman, Analysis of distances between inclusions in finite one-dimensional binary stochastic materials. Paper presented at International Conference on Mathematics, Computational Methods and Reactor Physics (M&C 2009), American Nuclear Society, New York, 3-7 May 2009.
- 21. C. Liang, W. Ji, F.B. Brown, Chord length sampling method for analyzing stochastic distribution of fuel particles in continuous energy simulations. Ann. Nucl. Energy **53**, 140-146 (2013). doi:10.1016/j.anucene.2012.09.013
- 22. K. Wang, Z.G. Li, D. She et al., RMC A Monte Carlo code for reactor core analysis. Ann. Nucl. Energy 82, 121-129 (2015). doi:10.1016/j.anucene.2014.08.048
- 23. S. Liu, K. Wang, Y. Chen, Development of Improved Chord-length Sampling Method and Its Application in Monte Carlo Simulation of Dispersion Fuel. Atomic Energy Science and Technology **54**, 1679-1684 (2020). (in Chinese)
- 24. D. She, Z.H. Liu, L. Shi, XPZ: Development of a lattice code for HTR. Ann. Nucl. Energy **97**, 183-189 (2016). doi:10.1016/j.anucene.2016.07.017
- 25. J. Li, D. She, L. Shi, An improved reactivity-equivalent physical transformation for treating FCM fuel with burnable poisons. Ann. Nucl. Energy. 121, 577-581 (2018). doi:10.1016/j.anucene.2018.08.024
- 26. Q.Q. Pan, N. An, T.F. Zhang et al., Single-step Monte Carlo criticality algorithm. Comput. Phys. Commun. 279, 108439 (2022). doi:10.1016/j.cpc.2022.108439

- 27. Z.X. Wu, D.C. Lin, D.X. Zhong, The design features of the HTR-10. Nucl. Eng. Des. **218**, 25-32 (2002). doi:10.1016/S0029-5493(02)00182-6
- 28. Z.Y. Zhang, Z.X. Wu, Y.L. Sun et al., Design aspects of the Chinese modular high-temperature gascooled reactor HTR-PM. Nucl. Eng. Des. **236**, 485-490 (2006). doi:10.1016/j.nucengdes.2005.11.024